AE 1254 – AIRCRAFT STRUCTURES -1

TWO MARK QUESTION & ANSWERS

UNIT: 1

STATICALLY DETERMINATE STRUCTURES

1. Explain with examples the statically determinate structures.

If the structure can be analyzed and the reactions at the support can be determined by using the equations of static equilibrium such as $\sum Fx = 0$, $\sum Fy = 0$ and $\sum M = 0$, then it is called as a statically determinate structure. Example: Simply supported beam, pin jointed truss or frame.

2. Differentiate Truss and Frame?

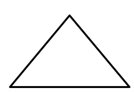
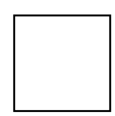
Truss	Frame
Truss is defined as number of members riveted together to carry the horizontal, vertical and inclined loads in equilibrium	Frame is defined as number of members welded together to carry the horizontal, vertical loads in equilibrium

3What are the types of frames.

Frames are classified into two types.

1. Perfect
2. Imperfect
   (i)Deficient frame
   (ii) Redundant frame

4. Differentiate the perfect and imperfect frames?

Sl.No.	Perfect frame	Imperfect frame
1.	Perfect frames have sufficient or enough members to carry the load	Imperfect frames have less or more members to carry the load than the required numbers.
2.	It satisfies the formula n= 2j-3	It does not satisfy the formula n= 2j-3
3.	Eg:Triangular frame  n=3, j = 3 n= 2j-3 3 =2x3 -3, 3 = 3	Eg: Square frame  n=4, j = 4 n= 2j-3 4 = 2x4 -3, $4 \neq 5$

Where, n = number of members, j= number of joints.

5. Differentiate the deficient frame and redundant frame?

Sl.No	deficient frame	redundant frame
1.	If the number of members are less than the required number of members. $n < 2j-3$	If the number of members are more than the required number of members $n > 2j-3$

6. Define Plane truss and Space truss? Give some examples.

A Plane truss is a two dimension truss structure composed of number of bars hinged together to form a rigid framework, all the members are lie in one plane. Eg: Roof truss in industries

A space truss is a three dimension truss structure composed of number of bars hinged together to form a rigid framework, the members are lie in different plane. Eg: Transmission line towers, crane parts.

7. What are the methods used to analyze the plane & space frames?

- ♦ Analytical method.
  1. Method of joints
  2. Method of sections (Method of moments)
  3. Tension coefficient method.
- ♦ Graphical method.

8. What are the assumptions made in the analyze of a truss?

1. In a frame or truss all the joints will be pin jointed.
2. All the loads will be acting at the joints only.
3. The self-weight of the members of the truss is neglected. Only the live load is considered
4. The frame is a perfect one.

9. What are conditions of equilibrium used in the method of joints? Why?

The conditions of equilibrium used in the method of joints are, $\sum Fx = 0$, $\sum Fy = 0$ .One of the assumption is all the joints are pin jointed, there is no moment. The equilibrium condition $\sum Mx = 0$ is not used.

10. What is cantilever truss? What is simply supported truss?

If anyone of the member of the truss is fixed and the other end is free, it is called a cantilever truss. There is no reaction force at the fixed end.

If the members of the truss are supported by simple supports, then it is called simply supported truss. Reaction forces are at the simply supported ends.

11. What are the hints to be followed while analyzing a cantilever truss using method of joints?

- ♦ There is no need to find the support reactions.
- ♦ The analysis is to be started from the free end where there is a maximum of two unknown forces, using the condition of equilibrium $\sum Fx = 0$ and $\sum Fy = 0$.
- ♦ All the members are assumed to be tensile
- ♦ Consider tensile forces as positive and compressive as negative.
- ♦ The force convention is, upward force assigns positive sign and downward force assigns negative sign.

12.What are the hints to be followed while analyzing a simply supported truss using method of joints?

- ♦ The support reactions are determined first.
- ♦ The analysis is to be started from the free end where there is a maximum of two unknown forces, using the condition of equilibrium $\sum Fx = 0$ and $\sum Fy = 0$.
- ♦ All the members are assumed to be tensile
- ♦ Consider tensile forces as positive and compressive as negative.

♦ The force convention is, upward force assigns positive sign and downward force assigns negative sign.

13. Give relation between the numbers of members and joints in a truss and explain its uses?

n= 2j-3, Where, n = number of members, j= number of joints. This relation is used to find the type of the frames. Perfect frame is only solved by method of joints.

14. A perfect frame consists of 7 members. Find the number of joints?

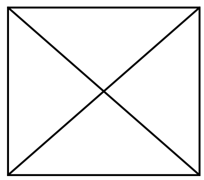
$n = 2j-3$

$7 = 2xj -3$

$2j = 7+3$

$J = 5$

15. Check the type of frame?



n = number of members =6

j = number of joints =4

$n = 2j-3$

$6 =2x4-3$

$6 \neq 5$

$6 > 5$

The given frame is an imperfect redundant frame.

16. What are the primary and secondary stresses in the analysis of a truss?

If the stresses are produced due to direct loads like tension, compression and torsion then the stresses are called primary stress. If the stresses are produced due to expansion, compression and temperature variation then the stresses are called secondary stress.

UNIT : II

STATICALLY INDETERMINATE STRUCTURES

1. Explain with examples the statically indeterminate structures.

If the forces on the members of a structure cannot be determined by using conditions of equilibrium ($\sum Fx = 0$, $\sum Fy = 0$, $\sum M = 0$), it is called statically indeterminate structures.

Example: Fixed beam, continuous beam.

2. Differentiate the statically determinate structures and statically indeterminate structures?

Sl.No	statically determinate structures	statically indeterminate structures
1.	Conditions of equilibrium are sufficient to analyze the structure	Conditions of equilibrium are insufficient to analyze the structure
2.	Bending moment and shear force is independent of material and cross sectional area.	Bending moment and shear force is dependent of material and independent of cross sectional area.
3.	No stresses are caused due to temperature change and lack of fit.	Stresses are caused due to temperature change and lack of fit.

3. Define: Continuous beam.

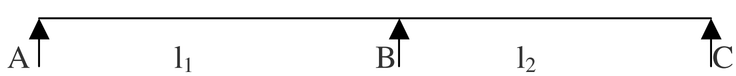
A Continuous beam is one, which is supported on more than two supports. For usual loading on the beam hogging (- ive) moments causing convexity upwards at the supports and sagging (+ ve) moments causing concavity upwards occur at mid span.

4. What are the advantages of Continuous beam over simply supported beam?

1. The maximum bending moment in case of continuous beam is much less than in case of simply supported beam of same span carrying same loads.

2. In case of continuous beam, the average bending moment is lesser and hence lighter materials of construction can be used to resist the bending moment.

5. Write down the general form of Clapeyron's three moment equations for the continuous beam.



$$M_a l_1 + 2 M_b l_2 + M_c l_2 = \left(\frac{6A_1 \overline{x_1}}{l_1} + \frac{6 A_2 \overline{x_2}}{l_2} \right)$$

where,

M_a = Hogging bending moment at A

M_b = Hogging bending moment at B

M_c = Hogging bending moment at C

l_1 = length of span between supports A,B

l_2 = length of span between supports B, C

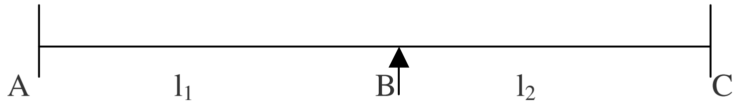
$\overline{x_1}$ = CG of bending moment diagram from support A

$\overline{x_2}$ = CG of bending moment diagram from support C

A_1 = Area of bending moment diagram between supports A,B

A_2 = Area of bending moment diagram between supports B, C

6. Write down the Clapeyron's three moment equations for the continuous beam with sinking at the supports.

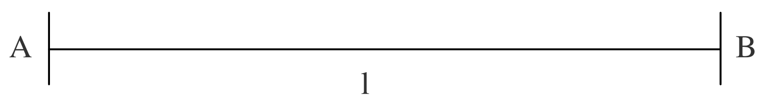


$$M_a l_1 + 2 M_b l_2 + M_c l_2 = \left(\frac{6A_1 \bar{x}_1}{l_1} + \frac{6 A_2 \bar{x}_2}{l_2} \right), \quad - 6EI \left(\frac{\delta_I}{l_1} + \frac{\delta_2}{l_2} \right)$$

where,

M_a = Hogging bending moment at A
M_b = Hogging bending moment at B
M_c = Hogging bending moment at C
l_1 = length of span between supports A,B
l_2 = length of span between supports B, C
$\bar{x}_1$ = CG of bending moment diagram from support A
$\bar{x}_2$ = CG of bending moment diagram from support C
A_1 = Area of bending moment diagram between supports A,B
A_2 = Area of bending moment diagram between supports B, C
δ_I = Sinking at support A with compare to sinking at support B
δ_2 = Sinking at support C with compare to sinking at support B

7. Write down the Clapeyron's three moment equations for the fixed beam

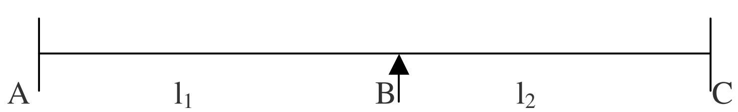


$$M_a + 2 M_b = \left(\frac{6A \bar{x}}{l^2} \right)$$

where,

M_a = Hogging bending moment at A
M_b = Hogging bending moment at B
l = length of span between supports A,B
$\bar{x}$ = CG of bending moment diagram from support A
A = Area of bending moment diagram between supports A,B

8. Write down the Clapeyron's three moment equations for the continuous beam carrying UDL on both the spans.



$$M_a l_1 + 2 M_b l_2 + M_c l_2 = \left(\frac{6A_1 \bar{x}_1}{l_1} + \frac{6 A_2 \bar{x}_2}{l_2} \right) = \frac{w_1 l_1^3}{4} + \frac{w_2 l_2^3}{4}$$

where,

M_a = Hogging bending moment at A
M_b = Hogging bending moment at B
M_c = Hogging bending moment at C
l_1 = length of span between supports A,B
l_2 = length of span between supports B, C

9. Give the values of ($6A_1 \bar{x}_1 / l_1$), ($6A_2 \bar{x}_2 / l_2$) values for different type of loading.

Type of loading	$6A_1 x_1 / l_1$	$6A_2 x_2 / l_2$
UDL for entire span	$wl^3 / 4$	$wl^3 / 4$
Central point loading	$(3/8)\ Wl^2$	$(3/8)\ Wl^2$
Uneven point loading	$(wa / l) /(l^2 – a^2)$	$(wb / l) /(l^2 – b^2)$

10. Give the procedure for analyzing the continuous beams with fixed ends using three moment equations?

The three moment equations, for the fixed end of the beam, can be modified by imagining a span of length l_0 and moment of inertia, beyond the support the and applying the theorem of three moments as usual.

11. Define: Moment distribution method.(Hardy Cross mrthod).

It is widely used for the analysis of indeterminate structures. In this method, all the members of the structure are first assumed to be fixed in position and fixed end moments due to external loads are obtained.

12. Define: Stiffness factor.

It is the moment required to rotate the end while acting on it through a unit rotation, without translation of the far end being

(i) Simply supported is given by $k = 3\ EI / L$

(ii) Fixed is given by $k = 4\ EI / L$

Where. E = Young's modulus of the beam material.
I = Moment of inertia of the beam
L = Beam's span length.

13. Define: Distribution factor.

When several members meet at a joint and a moment is applied at the joint to produce rotation without translation of the members, the moment is distributed among all the members meeting at that joint proportionate to their stiffness.

Distribution factor = Relative stiffness / Sum of relative stiffness at the joint

If there is 3 members, Distribution factors = $\dfrac{k_1}{k_1 + k_2 + k_3}$, $\dfrac{k_2}{k_1 + k_2 + k_3}$, $\dfrac{k3}{k_1 + k_2 + k_3}$

14. Define: Carry over moment and Carry over factor.

Carry over moment: It is defined as the moment induced at the fixed end of the beam by the action of a moment applied at the other end, which is hinged. Carry over moment is the same nature of the applied moment.

Carry over factor (C.O) : A moment applied at the hinged end B " carries over" to the fixed end A, a moment equal to half the amount of applied moment and of the same rotational sense. C.O =0.5

15. Define Flexural Rigidity of Beams.

The product of young's modulus (E) and moment of inertia (I) is called Flexural Rigidity (EI) of Beams. The unit is N mm^2.

16. Define: Constant strength beam.

If the flexural Rigidity (EI) is constant over the uniform section, it is called Constant strength beam.

17. Define: Composite beam.

A structural member composed of two or more dissimilar materials jointed together to act as a unit. The resulting system is stronger than the sum of its parts. The composite action can better utilize the properties of each c constituent material.

Example : Steel- Concrete composite beam, Steel- Wood beam.

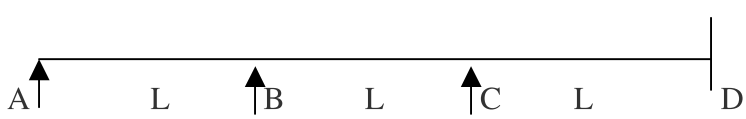
18. Write the two conditions for the analysis of composite beam.

(i).Strain (Stress x E) in all the material are same ($e_1 = e_2$)

($e_1 = (Pl_1 / A_1E_1)$; $e_2 = (Pl_2 / A_2E_2)$)

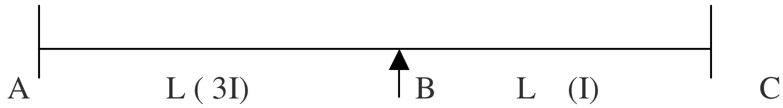
(ii).The total load = $P_1 + P_2 + P_3$ (P_1 = Stress x area).

19. Find the distribution factor for the given beam.



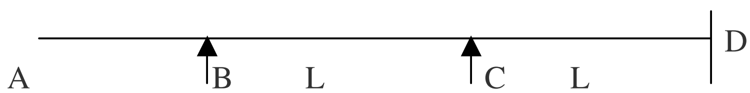
Join	Member	Relative stiffness	Sum of Relative stiffness	Distribution factor
A	AB	4EI / L	4EI / L	(4EI / L) / (4EI / L) = 1
B	BA	3EI /L	3EI /L + 4EI / L = 7EI / L	(3EI / L) / (7EI / L)= 3/7
	BC	4EI / L		(4EI / L) / (7EI / L) = 4/7
C	CB	4EI / L	4EI / L + 4EI / L =8EI / L	(4EI / L) / (8EI / L) =4/8
	CD	4EI / L		(4EI / L) / (8EI / L)= 4/8
D	DC	4EI / L	4EI / L	(4EI / L)/ (4EI / L) = 1

20. Find the distribution factor for the given beam.



Join	Member	Relative stiffness	Sum of Relative stiffness	Distribution factor
A	AB	4E (3I) / L	12EI / L	(12EI / L) / (12EI / L) = 1
B	BA BC	4E(3I) /L 4EI / L	12EI /L + 4EI / L = 16EI / L	(12EI / L) / (16EI / L)= 3/4 (4EI / L) / (16EI / L) = 1/4
C	CB	4EI / L	4EI / L	(4EI / L) / (4EI / L) =1

21. Find the distribution factor for the given beam.



Join	Member	Relative stiffness	Sum of Relative stiffness	Distribution factor
B	BA BC	0(no support) 4EI / L	4EI / L	0 (4EI / L) / (4EI / L) =1
C	CB CD	3EI / L 4EI / L	3EI /L + 4EI / L = 7EI / L	(3EI / L) / (7EI / L)= 3 / 7 (4EI / L) / (7EI / L) = 4 / 7
D	DC	4EI / L	4EI / L	(4EI / L) / (4EI / L) =1

UNIT : III

ENERGY METHODS

1.Define: Strain Energy

When an elastic body is under the action of external forces the body deforms and work is done by these forces. If a strained, perfectly elastic body is allowed to recover slowly to its unstrained state. It is capable of giving back all the work done by these external forces. This work done in straining such a body may be regarded as energy stored in a body and is called strain energy or resilience.

2. Define: Proof Resilience.

The maximum energy stored in the body within the elastic limit is called Proof Resilience.

3. Write the formula to calculate the strain energy due to axial loads (tension).

$$U = \int \frac{P^2}{2AE} dx \qquad \text{limit } 0 \text{ to } L$$

Where,

P = Applied tensile load.
L = Length of the member
A = Area of the member
E = Young's modulus.

4. Write the formula to calculate the strain energy due to bending.

$$U = \int \frac{M^2}{2EI} dx \qquad \text{limit } 0 \text{ to } L$$

Where,

M = Bending moment due to applied loads.
E = Young's modulus
I = Moment of inertia

5. Write the formula to calculate the strain energy due to torsion

$$U = \int \frac{T^2}{2GJ} dx \qquad \text{limit } 0 \text{ to } L$$

Where, T = Applied Torsion
G = Shear modulus or Modulus of rigidity
J = Polar moment of inertia

6.Write the formula to calculate the strain energy due to pure shear

$$U = K \int \frac{V^2}{2GA} dx \qquad \text{limit } 0 \text{ to } L$$

Where, V= Shear load
G = Shear modulus or Modulus of rigidity
A = Area of cross section.
K = Constant depends upon shape of cross section.

7. Write down the formula to calculate the strain energy due to pure shear, if shear stress is given.

$$U = \frac{\tau^2 V}{2G}$$

Where, τ = Shear Stress
G = Shear modulus or Modulus of rigidity
V = Volume of the material.

8. Write down the formula to calculate the strain energy , if the moment value is given

$$U = \frac{M^2 L}{2EI}$$

Where, M = Bending moment
L = Length of the beam
E = Young's modulus
I = Moment of inertia

9. Write down the formula to calculate the strain energy , if the torsion moment value is given.

$$U = \frac{T^2 L}{2GJ}$$

Where, T = Applied Torsion
L = Length of the beam
G = Shear modulus or Modulus of rigidity
J = Polar moment of inertia

10. Write down the formula to calculate the strain energy, if the applied tension load is given.

$$U = \frac{P^2 L}{2AE}$$

Where,
P = Applied tensile load.
L = Length of the member
A = Area of the member
E = Young's modulus.

11. Write the Castigliano's first theorem.

In any beam or truss subjected to any load system, the deflection at any point is given by the partial differential coefficient of the total strain energy stored with respect to force acting at a point.

$$\delta = \frac{\partial U}{\partial P}$$

Where,
δ = Deflection
U= Strain Energy stored
P = Load

12. What are uses of Castigliano's first theorem?
1.To determine the deflection of complicated structure.
2. To determine the deflection of curved beams, springs.

13.Define : Maxwell Reciprocal Theorem.

In any beam or truss the deflection at any point 'A' due to a load 'W' at any other point 'C' is the same as the deflection at 'C' due to the same load 'W' applied at 'A'.

$$\delta_A = \delta_C$$

14. Define: Unit load method.

The external load is removed and the unit load is applied at the point, where the deflection or rotation is to found.

15. Give the procedure for unit load method.

1. Find the forces P1, P2, ……. in all the members due to external loads.
2. Remove the external loads and apply the unit vertical point load at the joint if the vertical deflection is required and find the stress.
3. Apply the equation for vertical and horizontal deflection.

16. Compare the unit load method and Castigliano's first theorem

In the unit load method, one has to analyze the frame twice to find the load and deflection. While in the latter method, only one analysis is needed.

17. Find the strain energy per unit volume, the shear stress for a material is given as 50 N/mm ^{2}. Take G= 80000 N/mm ^{2}.

$$U = \frac{\tau^2}{2G} \quad \text{per unit volume}$$

$= 50^2 / (2 \times 80000)$

$= 0.015625$ N/mm ^{2}. per unit volume.

18. Find the strain energy per unit volume, the tensile stress for a material is given as 150 N/mm ^{2}. Take E = 2 x10 N/mm ^{2}.

$$U = \frac{f^2}{2E} \quad \text{per unit volume}$$

$= (150)^2 / (2 \times (2\times 10^2)$

$= 0.05625$ N/mm ^{2}. per unit volume.

19.Define : Modulus of resilience.

The proof resilience of a body per unit volume. (ie) The maximum energy stored in the body within the elastic limit per unit volume.

20. Define : Trussed Beam.

A beam strengthened by providing ties and struts is known as Trussed Beams.

UNIT : IV

COLUMN

1.Define: Column

A structural member, subjected to an axial compressive force is called a strut. A strut may be horizontal, inclined or even vertical. But a vertical strut used in buildings or frames is called a column and whose lateral dimensions are small as compared to its height.

2. What are the types of column failure?

1. Crushing failure:

The column will reach a stage, when it will be subjected to the ultimate crushing stress, beyond this the column will fail by crushing The load corresponding to the crushing stress is called crushing load. This type of failure occurs in short column.

2. Buckling failure:

This kind of failure is due to lateral deflection of the column. The load at which the column just buckles is called buckling load or crippling load or critical load. This type of failure occurs in long column.

3. What is slenderness ratio (buckling factor)? What is its relevance in column?

It is the ratio of effective length of column to the least radius of gyration of the cross sectional ends of the column.

Slenderness ratio = l eff / r

l eff = effective length of column

r = least radius of gyration

Slenderness ratio is used to differentiate the type of column. Strength of the column depends upon the slenderness ratio, it is increased the compressive strength of the column decrease as the tendency to buckle is increased.

4. What are the factors affect the strength column?

1.Slenderness ratio

Strength of the column depends upon the slenderness ratio, it is increased the compressive strength of the column decrease as the tendency to buckle is increased.

2. End conditions: Strength of the column depends upon the end conditions also.

5. Differentiate short and long column

Short column	Long column
1. It is subjected to direct compressive stresses only.	It is subjected to buckling stress only.
2. Failure occurs purely due to crushing only.	Failure occurs purely due to bucking only.
3. Slenderness ratio is less than 80	Slenderness ratio is more than 120.
4.It's length to least lateral dimension is less than 8. (L / D ‹ 8)	It's length to least lateral dimension is more than 30. (L / D › 30)

6. What are the assumptions followed in Euler's equation?

1. The material of the column is homogeneous, isotropic and elastic.
2. The section of the column is uniform throughout.
3. The column is initially straight and load axially.
4. The effect of the direct axial stress is neglected.
5. The column fails by buckling only.

7. What are the limitations of the Euler's formula?

1. It is not valid for mild steel column. The slenderness ratio of mild steel column is less than 80.
2. It does not take the direct stress. But in excess of load it can withstand under direct compression only.

8. Write the Euler's formula for different end conditions.

1. Both ends fixed.

$$P_E = \frac{\pi^2 EI}{(0.5L)^2}$$

2. Both ends hinged

$$P_E = \frac{\pi^2 EI}{(L)^2}$$

3. One end fixed ,other end hinged.

$$P_E = \frac{\pi^2 EI}{(0.7L)^2}$$

4. One end fixed, other end free.

$$P_E = \frac{\pi^2 EI}{(2L)^2}$$

L = Length of the column

9. Define: Equivalent length of the column.

The distance between adjacent points of inflection is called equivalent length of the column. A point of inflection is found at every column end, that is free to rotate and every point where there is a change of the axis. ie, there is no moment in the inflection points. (Or)

The equivalent length of the given column with given end conditions, is the length of an equivalent column of the same material and cross section with hinged ends , and having the value of the crippling load equal to that of the given column.

10. What are the uses of south well plot? (column curve).

The relation between the buckling load and slenderness ratio of various column is known as south well plot.

The south well plot is clearly shows the decreases in buckling load increases in slenderness ratio.

It gives the exact value of slenderness ratio of column subjected to a particular amount of buckling load.

11. Give Rakine's formula and its advantages.

$$P_R = \frac{f_C\ A}{(1 + a\,(l_{eff}\ /\ r)^2\ )}$$

where, P_R = Rakine's critical load
f_C = yield stress
A = cross sectional area
a = Rakine's constant
l_{eff} = effective length
r = radius of gyration

In case of short column or strut, Euler's load will be very large. Therefore, Euler's formula is not valid for short column. To avoid this limitation, Rankine's formula is designed. The Rankine's formula is applicable for both long and short column.

12. Write Euler's formula for maximum stress for a initially bent column?

$$\sigma\ max = P\ /A + \ (\ M\ max\ /\ Z\ )$$

$$= P/\ A\ + \frac{P\ a}{(\ 1-(\ P\ /\ P_E\ ))Z}$$

Where, P = axial load
A = cross section area
P_E = Euler's load
a = constant
Z = section modulus

13. Write Euler's formula for maximum stress for a eccentrically loaded column?

$$\sigma\ max = P\ /A + \ (\ M\ max\ /\ Z)$$

$$= P/\ A + \frac{P\ e\ Sec\ (l\ eff\ /\ 2\ )\ \sqrt{(P/EI)}}{(1-(P\ /\ P_E\ )\ )\ Z}$$

Where, P = axial load
A = cross section area
P_E = Euler's load
e = eccentricity
Z = section modulus
EI = flexural rigidity

14. What is beam column? Give examples.

Column having transverse load in addition to the axial compressive load are termed as beam column.

Eg : Engine shaft, Wing of an aircraft.

15. Write the expressions for the maximum deflection developed in a beam column carrying central point load with axial load, hinged at both ends.

$$\delta_{max} = \delta_O / (1 - (P/P_E))$$

where, $\delta_O = QL^3 / 48EI$

Q = central point load

P = axial load

P_E = Euler's load

16. Write the expressions for the maximum bending moment and max. stress developed in a beam column carrying central point load hinged at both ends.

$$M_{max} = M_O \frac{[1 - 0.18 (P/P_E)]}{[1 - (P/P_E)]}$$

$$\sigma_{max} = P/A + (M_{max} / Z)$$

where,

$M_O = QL / 4$

Q = central point load

P = axial load

P_E = Euler's load

Z = section modulus

17. Write the expressions for the maximum deflection developed in a beam column carrying uniformly distributed load with axial load, hinged at both ends.

$$\delta_{max} = \delta_O / (1 - (P/P_E))$$

where,

$\delta_O = 5w L^4 / 384EI$

w = uniformly distributed load / m run.

P = axial load

P_E = Euler's load

18. Write the expressions for the maximum bending moment and max, stress developed in a beam column carrying uniformly distributed load with axial load, hinged at both ends.

$$M_{max} = M_O \frac{[1 + 0.03 (P/P_E)]}{[1 - (P/P_E)]}$$

$$\sigma_{max} = P/A + (M_{max} / Z)$$

where,

$M_O = wL^2 / 8$

w = uniformly distributed load / m run

P = axial load

P_E = Euler's load

Z = section modulus

19. Write the expressions for the deflection developed in a beam column carrying several point loads at different distances with an axial load, hinged at both ends.

$$y = \frac{\sin kx}{Pk \sin kL} [Q_1 \sin kc_1 + Q_2 \sin kc_2 + Q_3 \sin kc_3 \ldots\ldots] - (x / PL) [Q_1 c_1 + Q_2 c_2 + Q_3 c_3 \ldots]$$

$$+ \frac{\sin k(L-x)}{Pk \sin kL} [Q_2 \sin k(L - c_2) + Q_3 \sin k(L - c_3) +] - ((L - x) / PL)) [Q_2 (L - c_2) + Q_3 (L - c_3)]$$

20. Write the general expressions for the maximum bending moment, if the deflection curve equation is given.

$$BM = - EI (d^2y / dx^2)$$

UNIT : V

THEORIES OF FAILURE

1. What are the types of failures?

 1. Brittle failure:

 Failure of a material represents direct separation of particles from each other, accompanied by considerable deformation.

 2. Ductile failure:

 Slipping of particles accompanied, by considerable plastic deformations.

2.List out different theories of failure

1. Maximum Principal Stress Theory. (Rakine's theory)
2. Maximum Principal Strain Theory. (St. Venant's theory)
3. Maximum Shear Stress Theory. (Tresca's theory or Guest's theory)
4. Maximum Shear Strain Theory. (Von –Mises- Hencky theory or Distortion energy theory)
5. Maximum Strain Energy Theory. (Beltrami Theory or Haigh's theory)

3. Define: Maximum Principal Stress Theory. (Rakine's theory)

According to this theory, the failure of the material is assumed to take place when the value of the maximum Principal Stress (σ_1) reaches a value to that of the elastic limit stress(f_y) of the material. $\sigma_1 = f_y$.

4. Define: Maximum Principal Strain Theory. (St. Venant's theory)

According to this theory, the failure of the material is assumed to take place when the value of the maximum Principal Stain (e_1) reaches a value to that of the elastic limit strain(f_y / E) of the material.

$$e_1 = f_y / E$$

In 3D, $e_1 = 1/E[\ \sigma_1 - (1/m)(\ \sigma_2 + \sigma_3)\ ] = f_y / E \rightarrow [\ \sigma_1 - (1/m)(\ \sigma_2 + \sigma_3)\ ] = f_y$

In 2D, $\sigma_3 = 0 \rightarrow$ $e_1 = 1/E[\ \sigma_1 - (1/m)(\ \sigma_2\ )\ ] = f_y / E \rightarrow [\ \sigma_1 - (1/m)(\ \sigma_2\ )\ ] = f_y$

5. Define : Maximum Shear Stress Theory. (Tresca's theory)

According to this theory, the failure of the material is assumed to take place when the maximum shear stress equal determined from the simple tensile test.

In 3D, $(\ \sigma_1 - \sigma_3) / 2 = f_y /2 \rightarrow (\ \sigma_1 - \sigma_3) = f_y$

In 2D, $(\ \sigma_1 - \sigma_2) / 2 = f_y /2 \rightarrow \sigma_1 = f_y$

6. Define : Maximum Shear Strain Theory (Von –Mises- Hencky theory or Distortion energy theory)

According to this theory, the failure of the material is assumed to take place when the maximum shear strain exceeds the shear strain determined from the simple tensile test.

In 3D, shear strain energy due to distortion $U = (1/ 12G)[\ (\ \sigma_1 - \sigma_2)^2 + (\ \sigma_2 - \sigma_3)^2 + (\ \sigma_3 - \sigma_1)^2\ ]$

Shear strain energy due to simple tension, $U = f_y^{\ 2} / 6G$

$$(1/12G)[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2] = f_y^2/6G$$

$$[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2] = 2f_y^2$$

In 2D, $$[(\sigma_1-\sigma_2)^2+(\sigma_2-0)^2+(0-\sigma_1)^2] = 2f_y^2$$

7. Define: Maximum Strain Energy Theory (Beltrami Theory)

According to this theory, the failure of the material is assumed to take place when the maximum strain energy exceeds the strain energy determined from the simple tensile test.

In 3D, strain energy due to deformation $U = (1/2E)[\sigma_1^2+\sigma_2^2+\sigma_3^2 - (1/m)(\sigma_1\sigma_2+\sigma_2\sigma_2+\sigma_2\sigma_2)]$

strain energy due to simple tension, $U = f_y^2/2E$

$$(1/2E)[\sigma_1^2+\sigma_2^2+\sigma_3^2 - (2/m)(\sigma_1\sigma_2+\sigma_2\sigma_2+\sigma_2\sigma_2)] = f_y^2/2E$$

$$[\sigma_1^2+\sigma_2^2+\sigma_3^2 - (2/m)(\sigma_1\sigma_2+\sigma_2\sigma_2+\sigma_2\sigma_2)] = f_y^2$$

In 2D, $$[\sigma_1^2+\sigma_2^2 - (2/m)(\sigma_1\sigma_2)] = f_y^2$$

8. What are the theories used for ductile failures?

1. Maximum Principal Strain Theory. (St. Venant's theory)
2. Maximum Shear Stress Theory. (Tresca's theory)
3. Maximum Shear Strain Theory. (Von –Mises- Hencky theory or Distortion energy theory)

9. Write the limitations of Maximum Principal Stress Theory. (Rakine's theory)

1. This theory disregards the effect of other principal stresses and effect of shearing stresses on other planes through the element.

2. Material in tension test piece slips along 45^0 to the axis of the test piece, where normal stress is neither maximum nor minimum, but the shear stress is maximum.

3.Failure is not a brittle, but it is a cleavage failure.

10. Write the limitations of Maximum Shear Stress Theory. (Tresca's theory).

This theory does not give the accurate results for the state of stress of pure shear in which the maximum amount of shear is developed (in torsion test).

11.Write the limitations of Maximum Shear Strain Theory.(Von –Mises- Hencky theory or Distortion energy theory).

It cannot be applied for the materials under hydrostatic pressure.

12. Write the limitations of Maximum Strain Energy Theory. (Beltrami Theory).

This theory does not apply to brittle materials for which elastic limit in tension and in compression are quite different.

13. Write the failure theories and its relationship between tension and shear.

1. Maximum Principal Stress Theory. (Rakine's theory) $\zeta_y = f_y$

2.Maximum Principal Strain Theory. (St. Venant's theory) $\zeta_y = 0.8\, f_y$

3. Maximum Shear Stress Theory. (Tresca's theory) $\zeta_y = 0.5\, f_y$

4.Maximum Shear Strain Theory (Von– Mises - Hencky theory or Distortion energy theory) $\zeta_y = 0.577\, f_y$

5. Maximum Strain Energy Theory. (Beltrami Theory) $\zeta_y = 0.817 f_y$.

14. Write the volumetric strain per unit volume. $f_y^2 / 2E$

20. Define : Octahedral Stresses

A plane, which is equally inclined to the three axes of reference, is called octahedral plane. The normal and shearing stress acting on this plane are called octahedral stresses.

$$\tau_{oct} = 1/3 \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

21. Define: Plasticity ellipse.

The graphical surface of a Maximum Shear Strain Theory (Von –Mises- Hencky theory or Distortion energy theory) is a straight circular cylinder. The equation in 2D is

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = f_y^2 \text{ which is called the Plasticity ellipse}$$